hours may well be continued thru the whole of an investigator's life, they will accomplish the object for which they are written.

A SPECTROSCOPIC METHOD OF DETERMINING STELLAR PARALLAX[1]

By Walter S. Adams.

The absolute magnitude of a star is defined as its apparent magnitude when reduced to unit distance, and may be calculated, when the distance is known, from the simple relationship

$$M = m + 5 + 5 \log \pi$$

In this equation M is the absolute magnitude, m the apparent magnitude, and π the parallax. It is evident that if some means exists by which M may be determined directly this expression will in turn provide a determination of the parallax.

The question whether the absolute magnitude of a star may not have a material influence upon the character of its spectrum is one which has been discussed to a considerable extent in recent years. It appears probable from investigations made at Mount Wilson that the intensity of the continuous spectrum is affected to a marked degree by the luminosity of the star; and such a result is rather to be expected when we consider that if two stars have closely the same type of spectrum but greatly different luminosities they probably differ greatly in size, in mass, and in depth of the atmospheres surrounding them. Such a marked difference in physical conditions would be almost certain to influence the intensity of the continuous spectrum, and we might even hope to find in the line spectrum of these stars certain variations in the intensity and character of such lines as are peculiarly sensitive to the conditions in the gases in which they find their origin.

[1] Adapted from a series of papers published in the *Proceedings of the National Academy of Sciences*.

An attempt to detect such lines was made by Hertzsprung who concluded that the strontium line at λ 4077 gave some indication of varying with the absolute magnitudes of the stars in whose spectra it appeared. An independent and more extensive investigation by Dr. Kohlschütter of the Mount Wilson stellar spectra resulted in the discovery of two or three such lines, and the results of a preliminary application of the criterion of line intensity to the determination of absolute stellar magnitudes were published in 1914. Since that time the work has been extended greatly with the aid of additional observational material.

A prime essential to beginning such an investigation is an accurate classification of the stellar spectra into the several types. For this purpose a method has been used which was devised in large measure by Dr. Kohlschütter previous to his departure from Mount Wilson. It will be described briefly in the succeeding paragraphs, and the results obtained from it will then be utilized in a continuation of the discussion of absolute magnitudes.

Method of Classification

The material available for classification purposes consists of several thousand photographs of stellar spectra taken with a one-prism slit spectrograph and the sixty-inch reflector. About two-thirds of these spectra are of types succeeding Fo. On most of the photographs the region of spectrum in best definition extends from λ 4200 to λ 4900. It includes, therefore, the two hydrogen lines Hγ and Hβ, the important calcium line at λ 4227, and some of the most prominent iron lines in the entire spectrum. Since the hydrogen lines show a rapid decrease in intensity with the successive types F, G, K and M, and form by far the most important criterion in the determination of spectral type, accurate determinations of their intensity relative to other lines in the spectrum are essential. Accordingly several adjacent iron lines have been selected which show but a moderate change of intensity in these types, and estimates are made on an arbitrary scale, extending from zero to ten, of the differences in intensity between the hydrogen lines and this selected list. The calcium line λ 4227 is also compared with Hγ in the types Fo to G5, beyond G5 the differences

becoming too great to provide satisfactory determinations. The list of pairs of lines finally adopted for classification purposes is given in Table I.

TABLE I

Line		Line	Element	Range of Type
Hγ	and	λ 4227	*Ca*	Fo to G5
Hγ	"	λ 4326	*Fe*	F3 to Ma
Hγ	"	λ 4352	*Fe, Mg*	Fo to Ma
Hγ	"	λ 4383	*Fe*	Fo to G5
Hγ	"	λ 4405	*Fe*	F3 to Ma
Hβ	"	λ 4872	*Fe*	Go to Ma
Hβ	"	λ 4957	*Fe*	Go to Ma

The scale of classification was adapted to the Harvard system by selecting a considerable number of stars for which Harvard determinations were available, and making estimates of the relative intensities of these pairs of lines in the stars selected. The values were then plotted against the average types of these stars, and smooth curves were drawn thru the several points. These curves provide the means of converting determinations of relative line intensity into determinations of spectral type. They are based upon stars of large proper motion alone, and the material may, therefore, be regarded as homogeneous in character.

To illustrate the use of these curves I have selected as examples the stars Groom. 3357, Piazzi 0h130, Groom. 145 and Lal. 19022. The estimated differences of intensity for these stars, as determined from three photographs of their spectra, are given in Table II.

TABLE II

	4226 / Hγ	4326 / Hγ	Hγ / 4352	Hγ / 4383	Hγ / 4405	Hβ / 4872	Hβ / 4957	Bands
Groom. 3357	0.0	−5.3	+7.0	+4.3	+7.0			
Pi 0h130		+3.7	+0.7		−1.3	+3.3	+1.0	
Groom. 145		+5.3	−2.0		−4.0	−0.2	−1.8	
Lal. 19022		+6.3	−3.3		−5.0	−1.3	−3.3	1

With the aid of tables constructed from the curves we obtain the following determinations of spectral type from the separate pairs of lines:

						Mean	Probable Error
Groom. 3357	F7	F7	F4	F6	F5	F6	±1.0
Pi 0h130	G7	G5	G6	G4	G5	G5	0.8
Groom. 145	K1	K1	K3	K3	K2	K2	0.6
Lal. 19022	K7	K7	Ma	K6	K6	K7	1.6

The average probable error of the determination of type for these four stars is ± 1.0, and this is about the value obtained for several hundred stars classified in this way. It is evident that the accuracy will be least when the lines compared differ greatly in intensity, as in the types Fo–F9 and K5–Ma, and greatest when the lines are of nearly equal intensity.

This simple method of classification may be recommended as being rapid of operation, and free from the difficulties connected with the comparison of separate photographs with one another. It requires the establishment of a scale of relative intensity estimates by the observer, but this is a very simple matter when the range employed is small. To some extent the scale will be dependent upon the dispersion of the spectrograph employed since several of the lines used are compound in character. We have found, however, that with the single-prism spectrograph at Mount Wilson the same reduction curves may be used successfully for photographs on which the linear dispersion varies from 16 to 90 angstrom units to the millimeter at the center of the spectrum.

Determination of Absolute Magnitude and Parallax

The method of classification outlined above provides a means of securing a list of stars of closely similar spectral types. From among these stars pairs are then selected for which the absolute brightness is very different, and the relative intensities of the spectral lines are then examined carefully. To illustrate the method we may take as an example the two stars 61^1 *Cygni* and α *Tauri.* The measured parallaxes of these stars are $0''.31$ and $0''.07$, respectively, and their apparent magnitudes are 5.6 and 1.1. Their absolute magnitudes as computed from the equation

$$M = m + 5 + 5 \log \pi$$

are 8.0 and 0.4; that is, the luminosity of α *Tauri* is over 1100 times as great as that of 61^1 *Cygni.* A comparison of the spectra of the two stars side by side in a Hartmann spectro-comparator shows several points of difference. Of these two are most important. The calcium line at λ 4455 is very strong in 61^1 *Cygni* and relatively weak in α *Tauri;* and the strontium line at λ 4216 is weak in 61^1 *Cygni* and strong in α *Tauri.*

That this difference in behavior depends upon physical conditions in the stars and is not merely accidental is made almost certain by solar investigations. The line λ 4455 of calcium is greatly strengthened in the spectrum of sun-spots and increases in intensity with reduction in temperature. The line λ 4216 of strontium, on the other hand is an enhanced line, that is, stronger in the spectrum of the spark than of the arc, and is probably a high temperature line. It is very prominent in the spectrum of the Sun's limb when photographed at eclipses, and also in the upper chromosphere. Numerous other smaller differences between the spectra of *α Tauri* and 61[1] *Cygni* all point in the same direction; the low temperature lines strengthened in sun-spots are stronger in 61[1] *Cygni;* the enhanced lines are stronger in *α Tauri*. It has seemed preferable, however, for two reasons to use only these two lines in the absolute magnitude investigation. First, because they show the effect most markedly; and second, because they appear to be influenced but slightly by closely adjoining lines which blend with them.

After the behavior of the two lines λ 4216 and λ 4455 had been examined in a large number of stars and the systematic differences had been found to persist thru a wide range of spectral type the attempt was made to establish a numerical relationship between their intensities and the absolute magnitudes of the stars in which they occur. As in the case of Hγ and Hβ used for classification purposes, lines were selected near λ 4216 and λ 4455, with which the intensities of these lines were compared, the differences of intensity being estimated on a scale extending from zero to ten. The pairs of lines finally adopted for all of this work are as follows:

Line	Element		Line	Element	
λ 4216	*Sr*	and	λ 4250	*Fe*	(a)
λ 4455	*Ca*		λ 4462	*Fe, Mn*	(b)
λ 4455	*Ca*		λ 4495	*Fe*	(c)

For convenience of reference these pairs of lines will be designated in the future as (a) (b) and (c). The value (a) = —2, for example, denotes that λ 4216 is estimated to be two units fainter than λ 4250.

As soon as the estimates had been completed a number of the stars with well-determined parallaxes were selected, their absolute magnitudes were computed, and curves were constructed in which

the observed differences of intensity for each pair of lines formed the abscissæ, and the absolute magnitudes the ordinates. The stars were divided into five groups according to spectral type and curves were drawn for each group. The groups are as follows:

Fo-F6; F7-G7; G8-K4; K5-K9; M.

The curves are so nearly straight lines in the case of the first three of these groups that straight lines have been adopted, the constants being derived by least square solutions. In the K5-K9 group the curve for (a) is a straight line but not for (b) or (c). It is probable that there are no straight lines in the M group, but this is very uncertain. The significance of a straight line is, of course, that the intensity of the line varies uniformly with the absolute magnitude.

The most serious difficulty in the construction of these curves is the scarcity of parallax determinations on stars of high luminosity. Parallax observers have confined their attention almost wholly to stars of large proper motion which promise to yield large parallaxes. With the aid, however, of the Yale observations on the very bright stars, and some most valuable determinations by Mr. van Maanen of the parallaxes of certain stars of small proper motion a number of stars of very high luminosity were selected upon which the lower portions of the curves could be based. Particularly in the cases of the K5-K9 and the M groups these portions of the curves are still most uncertain, and must be adjusted with the aid of additional parallax observations when they become available.

The list of formulæ derived for the several groups is given in Table III. The equations are from my own observations. A similar list, in which the constants differ slightly, has been obtained from the determinations of Miss Burwell, who has carried out a complete series of estimations of the line intensities in these stars. In the formulæ M is the absolute magnitude, and Δ the estimated difference of intensity for each of the pairs of lines.

TABLE III

	(a)	(b)	(c)
Fo–F7	$M = -1.9\ \Delta + 5.2$	$M = +2.0\ \Delta + 3.3$	$M = +2.8\ \Delta - 2.6$
F3–G7	$M = -1.4\ \Delta + 4.9$	$M = +2.0\ \Delta + 3.3$	$M = +2.8\ \Delta - 2.6$
G3–K4	$M = -1.6\ \Delta + 4.7$	$M = +1.6\ \Delta + 5.1$	$M = +2.3\ \Delta - 0.3$
K5–K9	$M = -1.8\ \Delta + 5.0$	Curve	Curve
M (low luminosity)	$M = -1.5\ \Delta + 6.9$	Curve	Curve

The equation and curves in the case of the M stars are applicable only to the stars of low luminosity. In the case of the Fo-F7 stars it is doubtful whether the equations given, which for (b) and (c) are the same as in the G group, are other than rough approximations. The enhanced lines in the early F stars are normally so prominent that it is not surprising that the method begins to break down at this point.

To illustrate the use of the formulæ and curves we may select as illustrations a few stars of different spectral types and magnitudes. These are collected in Table IV. The classification is from Mount Wilson determinations.

TABLE IV

STAR	Mag	Type	Δ			M				Parallax	
			(a)	(b)	(c)	(a)	(b)	(c)	Mean	Comp.	Obs.
Pi 10^h96...	7.6	F5	−0.7	+0.7	+3.0	+ 6.5	4.7	5.8	5.7	+0″.04	+0″.04
Sun.......	...	G0	−0.5	+0.5	+3.0	+ 5.6	4.3	5.8	5.2	....	....
Lal. 38287.	7.2	G5	−1.8	+1.5	+3.5	+ 7.4	+6.3	+6.2	7.3	+0.10	+0.09
α Arietis...	2.2	K0	+2.5	−2.4	+0.2	+ 1.0	1.3	+0.2	0.8	+0.05	+0.09
α Tauri....	1.1	K5	+3.0	−2.0	+0.5	− 0.4	+1.9	+0.5	0.7	+0.08	+0.07
61^2 *Cygni*..	6.3	K8	−1.8	+5.8	+7.7	+ 8.2	9.3	8.9	8.8	+0.32	+0.31
Groom. 34.	8.2	Ma	−2.2	+6.8	+9.2	+10.2	10.5	10.4	10.4	+0.28	+0.28

The parallaxes are computed from the absolute magnitudes by the formula, to which reference has already been made,

$$5 \log \pi = M - m - 5.$$

The results are given in the next to the last column of the table, and the measured parallaxes in the final column.

It is evident that in the case of the stars whose absolute magnitudes, as computed from the measured parallaxes, have been used in the derivation of the relationship between line intensity and absolute magnitude, the mean values of the magnitude will necessarily be identical with those derived from the formulæ. The agreement of the measured and the computed parallaxes of the individual stars, however, serves as important evidence bearing on the validity of the method.

In two tables published in detail in the *Proceedings* are collected all of the stars with measured parallaxes equal to or exceeding

$+0''.05$ for which we have spectral observations. The first contains the stars used in the derivation of the curves; but in the second the results are quite independent and serve as a rigorous test of the value of this means of computing stellar parallaxes. The average deviation, taken without regard to sign, between the observed and the computed values for the stars of the first table is $0''.024$; for those of the second table it is $0''.026$. The deviations are distributed as follows:

First Table.....$0-0''.02$ (37), $0''.02-0''.04$ (16), $>0''.04$ (5)
Second Table....$0-0''.02$ (43), $0''.02-0''.04$ (11), $>0''.04$ (12)

There is but one deviation exceeding $0''.1$ in the 124 stars contained in the two tables. There seems to be no marked systematic difference between the observed and the computed parallaxes; the former average somewhat larger, but this is due mainly to a few large discrepancies.

There are 25 stars with measured negative parallaxes for which we have made spectrum determinations. The largest value for any one of these stars as computed from the line intensities is $+0''.08$; the average value for all is $+0''.03$. The spectrum method, of course, gives no negative parallaxes.

It seems reasonable to conclude from these results that the method of computing absolute magnitudes and parallaxes from the variation of the intensities of lines in stellar spectra is capable of yielding results of a very considerable degree of accuracy. Especially in the K and M type stars of low luminosity the line variations are so great that such stars may be recognized from a mere inspection of the spectrum. Stars, for example, like 61 *Cygni*, Groom. 34 and Krüger 60 bear very evident marks of their intrinsic faintness in the remarkable intensity of the low temperature calcium lines in their spectra.

An important gain in the value of this method of determining stellar magnitudes and distances should result from an increase in the number of measured parallaxes of bright stars of small proper motion. Such stars will on the average prove to be very luminous, and as already stated the portion of the curves connecting line intensity with absolute magnitude is subject to much more uncer-

tainty in the case of the very high luminosity stars than in any of the others. It is probable that after such a revision has been made the method will find its most important application as a means of distinguishing these giant stars in the stellar system.

Mount Wilson Solar Observatory,
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THE DIURNAL VARIATION OF THE REFRACTION.

By R. H. Tucker.

The question of diurnal variation in the amount of the atmospheric refraction is not entirely a new one. But there are still points of novelty in its consideration, and the references to the subject are mostly rather vague, with but little data of definite value. It has been conceded that each observing station may require special treatment, and this is especially likely for one of the elevation of Mount Hamilton. Every observation of a standard star, made here, has been reduced in such a manner as to present, in the derived value of the latitude, a contribution to the refraction correction. For many years these values agree in giving a smaller value of the refractions during the night observations, than that of the Poulkova constant, which is already considerably smaller than that of Bessel, long adopted for meridian work in the large observatories of the world.

In the preliminary reductions of the fundamental work, carried on here from 1905 to 1908, it became evident that the value of the refraction in daytime observing was different from that of the night hours, and the subject was thoroly discussed, for the years 1905 and 1906 in *L. O. Bulletin* No. 231, published in April, 1913.

The variation there derived was based upon the combination of the extreme southern stars, and north stars at lower culmination, in two programs, differing twelve hours in right ascension. The separate stars, up to 80° zenith distance south, and 83° north, were also treated individually, for comparison of the effect of spectral type. No dependence upon type, or magnitude, or zenith distance,